An Experiment for Dilation Property in Ambiguity*

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Abstract

In multi-period decision-making under ambiguity, updating a prior can lead to posterior dilation after information. We explore the observability of such dilation. While a theory suggests dilation occurs in ambiguity-averse individuals, we found dilation only in ambiguity-seeking subjects. Additionally, our results suggest that introducing information may lead to joint probability formation.

Keywords: Updating; Dilation; Joint ambiguity; Independent ambiguity
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1. Introduction

When a decision maker faces an ambiguous situation in multiple periods, the set of conditional probabilities can become wider than in previous periods (Nishimura and Ozaki 2017, Kato et al. 2019, Shishkin and Ortoleva 2023). This phenomenon is referred to as the 'dilation property.' Among several applications (Bose and Renou 2014, Beasuchene, Li and Li 2019), Shishkin and Ortoleva (2023) test the dilation property by asking subjects for the certainty equivalents of bets on colors in urns before and after ambiguous information is revealed. They find that the dilation property is observed only in ambiguity-seeking individuals. Our paper aims to study whether the dilation property truly exists across a wide range of ambiguity attitudes.

We measure ambiguity preferences and observe dilation as follows. We define ambiguity preferences through the certainty equivalent of a lottery to bet on a "risky urn R that contains twenty blue and twenty white balls" and the certainty equivalent of a lottery to bet on an "ambiguous urn A containing forty balls, with an unknown composition of blue and white balls." We then compare the certainty equivalent of betting on a color drawn from an ambiguous urn without any prior information to the certainty equivalent of betting on the same urn after obtaining information about the color of one drawn ball, which is then returned to the urn. If this difference is positive, we define it as dilation.

2. Theory and Experiment

In this section, we explain the theoretical implications and their application in the experiment, along with the supporting mathematical framework. Let the prize be JPY 1500, if the bet on the color of a ball drawn from an urn is correct and let the prize be zero otherwise. Consider the urn that contains balls that could be either blue or white. Formally envisage drawing a ball consecutively twice from the urn in which the first ball is returned before the second draw. Then the state space can be described as Ω = {BB;BW;WB;WW} and its partition is denoted by {E1;E2} for E1 := {BB; BW} (a blue ball is drawn first), E2 := {WB; WW} (a white ball is drawn first). We describe the choices of a decision maker by employing the idea of "ε-contamination" (Nishimura and Ozaki 2017, Kato et al. 2019).

Let p_0 denotes a principal probability on Ω that decision makers specify with $(1-\varepsilon)$ 0% confidence. Define the ε-contamination of p^0 as

$$\{p^0\}^{\varepsilon} := \{\ (1-\varepsilon)\,(p_1^0,\ p_2^0,\ p_3^0,\ p_4^0) + \ \varepsilon\ (q_1, q_2, q_3, q_4) \mid (q_1, q_2, q_3, q_4) \text{ is any probability on } \Omega\ \}$$

The set of first marginal probabilities is written by

$$\wp_1 := \{(p_1 + p_2, p_3 + p_4) \ \mid ((p_1, p_2, p_3, p_4) \ \in \ \{p^0\}^{\varepsilon}\}$$

and denotes the set of conditional probabilities by

$$\wp_{|E_i} := \left\{\frac{p_i}{p_i+p_{i+1}}, \frac{p_{i+1}}{p_i+p_{i+1}} \mid \ (p_1, p_2, p_3, p_4) \ \in \{p^0\}^{\varepsilon}\right\} \ \text{ for } \quad i = 1,\ 2.$$

Let u : R $\rightarrow$ R be a utility function, and $u(1500)$ denotes the utility derived from winning the bet, while u(0) represents the utility from losing, given the assigned color in the bet.

2.1 The Risky Urn R

Consider the urn consisting of balls that could be either blue or white, and the composition is known, with twenty blue and twenty white balls. When the winning bet is the case that a blue ball is drawn, the certainty equivalent value of this risky urn (denoted as urn R) is expressed as:

$$(p_1^0 + p_2^0)u(1500) + (p_3^0 + p_4^0)u(0) \qquad \text{(R)}$$

2.2 The Single Ambiguous Urn A

Consider an urn containing a total of forty balls, either blue or white, where the composition is unknown. Suppose that blue is the winning color. For ambiguity-averse subjects, we have certainty equivalent of ambiguous urn A:

$$\min_{(p1\,+\,p2;\,p3\,+\,p4)\,\in\{p^0\}^{\varepsilon}|}[\,(1-\varepsilon)\,(p_1^0+p_2^0)u(1500)\,+\,\,(1-\varepsilon)\,(p_3^0\,+\,p_4^0+\varepsilon)u(0)\,]\,\text{ =:u(A)}$$

Since it holds that

$$(1-\,\varepsilon)\big(p_1^0\,+\,p_2^0\big)=\min_{(q1,q2,q3\,,q4)}\big[(1-\varepsilon)\big(p_1^0\,+\,p_2^0\big)+\,\varepsilon\big(q_1+\,q_2\big)\big],$$

which is attained by $(q_1, q_2) = (0, 0)$ and $q_3 + q_4 = 1$, we also have

$$(1\,-\,\varepsilon)\big(p_3^0\,+\,p_4^0+\varepsilon\big)=\max_{(q1,q2,q3\,,q4)}\big[(1\,-\,\varepsilon)\big(p_3^0\,+\,p_4^0\big)+\,\varepsilon\big(q_3+\,q_4\big)\big],$$

where $(q_3, q_4) = (1, 0)$ or $(0, 1)$ and $q_1 + q_2 = 0$.

For ambiguity-seeking subjects, the minimum is replaced with the maximum, and similar reasoning follows.

2.3 The Ambiguous Urns after Information (Urn A_b and A_w)

We consider a situation where the color of the ball drawn from an ambiguous urn is observed and then returned to the urn. After determining the ambiguous urn, which consists of either blue or white balls, the experimenter picks a ball and shows its color to the subjects. The ball is then returned to the urn, and another ball is drawn. The state space is now represented as Ω = {BB, BW, WB, WW}, with partitions E1 = {BB, BW} and E2 = {WB, WW}. The subjects are paid based on the result of the second draw, and the details of the bet are explained before the experiment. For ambiguity-averse subjects, the certainty equivalent value of A_b (the ambiguous urn after a blue ball is drawn) and that of A_w (after a white ball is drawn) are described as follows:

$$\min_{\left(\frac{p_1}{p_1+p_2},\frac{p_2}{p_1+p_2}\right)\in\wp|E_1}\left(\frac{p_1}{p_1+p_2}u(1500),\frac{p_2}{p_1+p_2}u(0)\right) \qquad \text{A\_b,}$$

$$\min_{\left(\frac{p_3}{p_3+p_4},\frac{p_4}{p_3+p_4}\right)\in\wp|E_2}\left(\frac{p_3}{p_3+p_4}u(1500),\frac{p_3}{p_3+p_4}u(0)\right) \qquad \text{A\_w}$$

where E_1 = {BB;BW} and E_1 = {WB;WW}.

With ε and principal probabilities, the certainty equivalent value of urn A_b is described as:

$$\min_{\left(\frac{p_1}{p_1+p_2},\frac{p_2}{p_1+p_2}\right)\in\wp|E_1}\left(\frac{p_1}{p_1+p_2}u(1500),\frac{p_2}{p_1+p_2}u(0)\right)$$

$$=\frac{(1-\varepsilon)p_1^0}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}u(1500)+\frac{(1-\varepsilon)p_2^0+\varepsilon}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}u(0)=:\mathrm{u(A\_b)}$$

Since it holds that $\frac{(1-\varepsilon)p_1^0}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}=\min_{(q_1,q_2)}\frac{(1-\varepsilon)p_1^0+\varepsilon q_1}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon(q_1+q_2)}$,

and $\frac{(1-\varepsilon)p_2^0+\varepsilon}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}=\max_{(q_1,q_2)}\frac{(1-\varepsilon)p_2^0+\varepsilon q_2}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon(q_1+q_2)}$. with (q_1, q_2) = (0, 1)

For ambiguity-seeking subjects, the objective function is defined by replacing the 'min' operator with 'max'. Thus, the certainty equivalent value of A_b is described as:

$$\max_{\left(\frac{p_1}{p_1+p_2},\frac{p_2}{p_1+p_2}\right)\in\wp|E_1}\left(\frac{p_1}{p_1+p_2}u(1500),\frac{p_2}{p_1+p_2}u(0)\right)$$

$$=\frac{(1-\varepsilon)p_1^0+\varepsilon}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}u(1500)+\frac{(1-\varepsilon)p_2^0}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}u(0)=:\mathrm{u(A\_b)}$$

The value of A_w for ambiguity averse individual and ambiguity seeking individual are calculated in the same manner. The following theorem states the prediction of the experimental results.

Theorem. Assume that the principal probability is symmetric: $p_i^0, i=1,2,3,4$ are the same for all ambiguous urns. For ambiguity averse subjects, we expect to observe *R > A > A_b = A_w* . *For* ambiguity seeking subjects, *R < A_b=A_w < A,* and for neutral subjects, *R =A= A_b = A_w.*

2.4 Eliciting Certainty Equivalent and Ambiguity Preference

In practice, to create a real ambiguous urn, if the process lacks transparency, subjects may feel deceived or assume a uniform distribution. To prevent this, we use the complex and unpredictable method from Hayashi and Wada (2010), where the urn's composition is decided after the experiment, in front of the subjects, and then the balls are placed. However, since the urn is created after all decisions are made, it is impossible to run a trial where a ball is drawn and returned before the urn is constructed, making it difficult to obtain the certainty equivalent after information is revealed. To address this, we ask: "If a blue ball is drawn from the ambiguous urn, what is your certainty equivalent for betting on the color after the blue ball is returned?"

. Ambiguity preference is determined by the sign of the difference between the certainty equivalent of the ambiguous urn A and that of the risky urn R: a negative difference indicates ambiguity aversion, a positive difference indicates ambiguity seeking, and a zero difference indicates ambiguity neutrality. The dilation property is observed through the sign of the difference between the second ambiguous bet on A_b (or A_w) and the first one on A. Ambiguity-averse subjects reveal a negative difference, ambiguity-seeking subjects exhibit a positive difference, and neutral subjects show no difference. To elicit the certainty equivalents of all lotteries for any urn, we use the BDM ()mechanism, a known truth-telling method.

3. The Results

Sixty-six undergraduate students participated in our experiment at Keiai University in January 2024. Each subject provided a value x (in JPY), and they acquired JPY y if $y \leq x$ (where y is a uniformly randomly determined value from [0, 1000].) The subjects faced a bet on drawing the blue color from the risky urn R the single ambiguous urn A, the ambiguous urns where the first draw was blue (A_b) or white (A_w). The experiment began only after all subjects correctly answered quizzes to confirm their understanding.

We measure ambiguity preferences by calculating the difference between the certainty equivalent of ambiguous urn A and that of risky urn R. Seventeen subjects (28%) gave negative responses (ambiguity-averse), 35 (53%) gave positive responses (ambiguity-seeking), and 14 (21%) gave zero responses (ambiguity-neutral). Theory predicts a positive correlation between the single ambiguous urn A and the ambiguous urn with information about the color of the first drawn ball, A_b and A_w.

Contrary to the theoretical predictions, an OLS regression (using Python) between A−R and A – A_b finds a negative relationship (t=−4.919; p=0.000) , and the relationship between A−R and A−A_w is also negative (t=−4.016; p=0.000p = 0.000). In both Figure 1 and 2, the horizontal axis (preference of ambiguity) represents A−R, and the vertical axis represents dilation. In Figure 1, dilation is calculated by A_b −A. In Figure 2, dilation is calculated by A_w −A.

To understand why we obtained results contrary to the theoretical predictions, we first consider the possibility that the subjects' principal probabilities were biased, while the prediction crucially depends on the symmetry of the principal probability. Only with symmetric principal probabilities, where $p_1^0 = p_2^0 = p_3^0 = p_4^0$, does the following condition hold :

$$(p_1^0 + p_2^0, p_3^0 + p_4^0) = (p_1^0/(p_1^0 + p_2^0)\ p_2^0/(p_1^0 + p_2^0)) = (p_3^0/(p_3^0 + p_4^0)\ p_4^0/(p_3^0 + p_4^0)).$$

However, in this experimental setup, we cannot know their principal probability.

Secondly, some subjects may integrate each bet and interpret the probabilities of consecutively drawn colors as being correlated, with the same colors being likely to be drawn. For example, if a subject believes that blue is more likely to be drawn after blue is observed ($p_1^0 > p_2^0$), they tend to report a higher certainty equivalent for A_b. Similarly, if blue is expected to be more likely drawn after white is observed ($p_3^0 > p_4^0$), they tend to report a higher certainty equivalent for A_w, and so on. This tendency may lead to results that are opposite to the theoretical predictions.

To test the hypothesis mentioned earlier, we examined the number of subjects whose answers were A_b = A_w =A: (no correlation), which was 30 (45%). The number of subjects who answered A_b<A_w was 14 (21%), and those who answered A_b>A_w were 22 (33%). On the other hand, among the subjects who answered A_b ≠ A_w the proportion of subjects who answered 'A − R ≥ 0 and min{A_b−A, A_w −A} ≥ 0' or 'A−R ≤ 0 and max{A_b −A. A_w −A } ≤ 0' is 9/36 (25%). We test whether the two proportions differ or not statistically by chi-square test. The result, with a domain value of 8.292 and a p-value of 0.003, indicates that the proportions differ significantly. The subjects who considered no correlation are unlikely to exhibit the opposite result compared to those who expected such correlation. Hence the opposite result to theory tends to arise from the subjects who considered correlated principal probability.

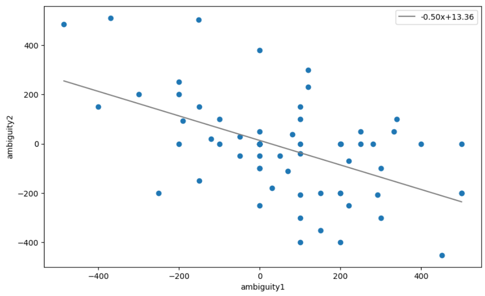

Fig.1 The relation between ambiguity preferences A− R (horizontal axis) and dilation A_b – A (vertical axis)

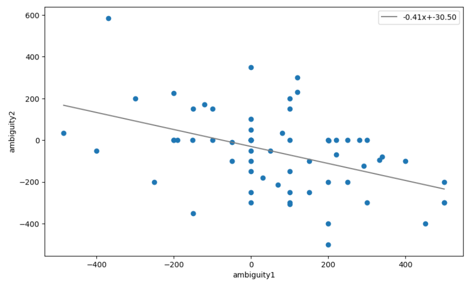

Fig.2 The relation between ambiguity preferences A− R (horizontal axis) and dilation A_w – A (vertical axis)

4. Conclusion

Our experimental results generally contradicted the predictions of the ε-contamination model. Information increased the value of the ambiguous urn for ambiguity-averse subjects and decreased it for ambiguity-seeking subjects. This suggests that subjects who integrated each bet perceived the probabilities of consecutive draws as correlated, thinking the same color was more or less likely to be drawn. However, when focusing only on subjects who did not believe there was a correlation between the first and second draws, the results aligned with the theory. Overall, subjects tended to integrate decisions and form joint probability assessments, which may indirectly manifest the dilation property. Further improvements and continuation of the experiment are needed to verify these findings.

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