Unstructured Bargaining Experiment on Three-person Cooperative Games

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Abstract

Cooperative game theory studies two problems: coalition formation and payoff distribution. To focus on the payoff distribution problem, the theory assumes the formation of the grand coalition, i.e., full cooperation among individuals that brings the greatest possible social surplus. However, if we consider the concept of the core, we cannot state that the grand coalition is always formed because a smaller group has an incentive to deviate from the grand coalition when the core is empty. On the other hand, the grand coalition will be formed in the games with the nonempty core where no smaller group can be better off by the deviation. Hence, there should be a difference in coalition formation between games with a nonempty core and games with an empty core. Additionally, the importance of communication among individuals is emphasized in textbook cooperative game theory. It is possible that whether these individuals can communicate with each other affects coalition formation. We hypothesized and examined these factors by a laboratory experiment in which subjects bargain about coalition formation and payoff distribution simultaneously. The bargaining protocol is characteristically unstructured, i.e., similar to the real bargaining situation. We find the following results. First, the grand coalition was not always formed though it brought the largest social surplus. Second, the grand coalition was more likely to be formed when the core was nonempty. Third, the possibility of communication among subjects induced their cooperation. Finally, the resulting payoffs reflected their power measured by the theory: stronger individuals take more profits.

JEL Classification: C71, C92

Keywords: laboratory experiment, unstructured bargaining, cooperative game, coalition formation, payoff distribution, the core

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1 Introduction

In the real world, doing a task by cooperating with others is usually more efficient than doing it alone, but cooperation needs to be agreed upon. For example, a business partnership among some firms reaches an agreement only when each of them believes that it brings more profits. This kind of issue is analyzed by cooperative game theory. The simplest case of the cooperative game is Nash's original bargaining problem, where there are only two individuals. In this case, the options for cooperation are simply to cooperate or not to cooperate. However, as the number of individuals increases, that of the options exponentially increases, and the problem becomes complicated. In cooperative game theory with three or more players, the most important issues are who to cooperate with and how to distribute the profits earned by the cooperation. These are called the coalition formation problem and payoff distribution problem.

Let us take a simple example. Consider three firms: A, B and C. The worth of the grand coalition is 120, and that of coalitions A and B, A and C, B and C is 100, 80, and 70, respectively. Singletons get nothing. Do they agree upon a payoff allocation $(40, 40, 40)$? We believe that the answer is no because A and B can be better off by making the coalition of A and B and implementing another payoff allocation $(50, 50, 0)$. Does this allocation become the final outcome? No, because A and C might refuse the allocation by making the coalition of A and C and proposing $(60, 0, 20)$. This bargaining is explained by the concept of the core. It is defined as a set of payoff allocations against which no subcoalition can be better off by deviating from the grand coalition with a feasible payoff allocation. The core is empty in the example above, so we can expect the grand coalition not to be formed even though the example is superadditive. On the other hand, we can also predict that the grand coalition will be formed when the core is nonempty. This issue is what we would like to examine by running a laboratory experiment.

2 Preliminaries

We first define the cooperative game with transferable utility. Let a pair (N, v) be a *game*. N is a finite player set, and v is a characteristic function. In our experiment, $|N| = 3$. Then there are three players A, B and C, so we describe this scenario as $N = \{A, B, C\}$. S, a subset of the player set, is called *coalition*. In this game, we have several possible coalitions that represent cooperation among the players: $\{A, B, C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A\}, \{B\}$ and $\{C\}$. The characteristic function v assigns a real number to each coalition and represents the profit gained by the coalition. We call $v(S)$ *worth of coalition* S. Let us suppose $v(N) = 120, v(AB)$[1] $= 90, v(AC) = 70, v(BC) = 50$ and $v(A) = v(B) = v(C) = 0$. This is one of the games which are implemented in our experiment.

[1] We write $v(AB)$ instead of $v(\{A, B\})$ for simplification.

Before defining the core, we first define *domination*. Let a vector $x = (x_A, x_B, x_C) \in \mathbb{R}^3$ be an *allocation*, which indicates who gets how much. In our experiment, the set of *feasible allocations* $\mathcal{A}$ is given as follows[2]:

$$\mathcal{A} = \{x \in \mathbb{R}^3_+ | x_A + x_B + x_C = v(N)\} \cup \{x \in \mathbb{R}^3_+ | x_A + x_B = v(AB), x_C = 0\} \cup$$
$$\{x \in \mathbb{R}^3_+ | x_A + x_C = v(AC), x_B = 0\} \cup \{x \in \mathbb{R}^3_+ | x_B + x_C = v(BC), x_A = 0\} \cup \{(0,0,0)\}.$$

Definition 1. *Take two feasible allocations $x, y \in \mathcal{A}$. Then x dominates y via coalition S if*

$$\sum_{i \in S} x_i = v(S) \quad \textit{and} \quad x_i > y_i \quad \forall i \in S.$$

Definition 2. *The core C(N,v) is defined as*

$$C(N, v) = \{x \in \mathcal{A} \mid \nexists y \in \mathcal{A} \;\; s.t. \;\; y \textit{ dominates } x \textit{ via some coalition } S \subseteq N.\}$$

We can find no feasible allocation dominating the core elements. In this sense, the core is expressed to be stable. However, if the core is empty, we are surely able to find a dominating allocation to any allocation in the grand coalition. Then, we can naturally say that the grand coalition will not be formed in such a situation.

3 Experimental Settings

In each round, the subjects played the role of a manager of virtual firms A, B and C. The 30 subjects were randomly divided into 10 groups of three and given one of the roles and information on the worth of the coalitions at the beginning of the round. They could not identify who belonged to their group.

Any player could make an offer at any moment of the round. The offer consists of two factors: a coalition and an allocation. After receiving the others' offer, the subjects can choose a reaction to it: to accept, to reject or to do nothing.

An offer reaches an agreement when all the players involved in the offered coalition accept it, and then the round ends. Nobody can change his or her mind after the agreement. When one of the two-person coalitions is formed, the one who is not involved in the coalition is considered to form the single person coalition. The time limit of each round is five minutes, and if the group uses it up without the agreement, the members of the group are regarded to form the single person coalition. After all the groups finish the round, the subjects are newly and randomly divided into the groups and given their roles to start the next round.

Table 1 shows the games the subjects played and their superadditivity and core. These games are the same as those in Nash et al. (2012).

We ran four treatments on table 2 to know whether there are two effects: one is the effect of communication and the other is that of learning.

[2] $\mathbb{R}^3_+$ is the non-negative quadrant of $\mathbb{R}^3$

Game	$v(N)$	$v(AB)$	$v(AC)$	$v(BC)$	$v(\cdot)$	Superadditivity	Core
1		120	100	90			
2		120	100	70			
3	120	120	100	50	0	Yes	empty
4		120	100	30			
5		100	90	70			
6		100	90	50			
7		100	90	30			
8	120	90	70	50	0	Yes	nonempty
9		90	70	30			
10		70	50	30			

Table 1: worth of coalitions, superadditivity and core

Chat Window	Game Order	
	1→10	10→1
Available	1A	1B
Not Available	2A	2B

Table 2: experimental treatments

4 Results

4.1 Coalition Formation

Now let us take a look at the results of the coalition formation. We have 800 observations: one group is one observation.

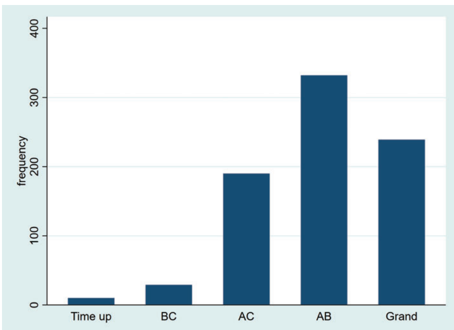

Figure 1: coalition formation

As shown in figure 1, coalition AB, which gives the highest profit of the two-person coalitions, was formed most frequently (332 times). The grand coalition is assumed to be formed in all superadditive games by the theory, but it was

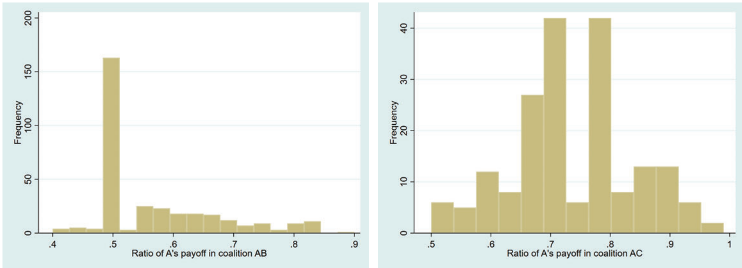

Figure 2: ratio of A's payoff in coalition AB and AC

actually formed only 239 times. The less valued coalitions, coalition AC and coalition BC, were formed 190 times and 29 times, respectively, and 10 out of 800 groups used up the five minutes without reaching an agreement.

4.2 Payoff Distribution

Let us focus on payoff distribution in the grand coalition. We have found that almost all the resulting allocations can be classified into three types: $x_A > x_B > x_C$, $x_A = x_B > x_C$ and $x_A = x_B = x_C$. We observed the first type 58 times, the second type 82 times, the third type 81 times and the others 18 times out of the 239 grand coalitions.

We can see that A frequently obtains approximately 50% of the worth of coalition AB, while A obtains from 70% to 80% of that of coalition AC.

5 Conclusion

First, although the cooperative game theory assumes that the grand coalition is always formed in all superadditive games, our result was different: only approximately 30% of the groups formed the grand coalition. The two-person coalition by A and B was more frequently formed than the grand coalition. Second, the grand coalition was more likely to be formed when the core was nonempty. When it is empty, any offer proposing the grand coalition can be dominated via a two-person coalition. On the other hand, elements of the core cannot be dominated by any two-person coalition. Hence, we can say that this finding is consistent with the concept of the core. Finally, the grand coalition was more likely to be formed when the chat window was available. Communication among individuals has been found to promote their cooperation, and the theory supposes that they can communicate with each other in bargaining. Our finding follows the preceding research and emphasizes the importance of communication.

We also found several items regarding payoff distribution. First, the resulting allocation reflected the players' bargaining power to some extent: the strongest player A takes the most, the second strongest player B takes the second most and the weakest player C takes the least. This result is supported by cooperative game theory, which suggests that the stronger a player is, the more he or she should take. Second, we observed the other two focal points: the equal division between A and B and the equal division among the three. Under the experimental bargaining scenario, A and B were thought to be equally strong even though A was stronger than B in the theory. When either the grand coalition or coalition AB was formed, A and B obtained the same amount of profit quite frequently. On the other hand, to form coalition AC, C had to give A more than half of the worth of coalition AB. We had the subjects play only 10 games, so if they play other games, the results might be different from ours.

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