Time-varying risk attitude and behavioral asset pricing

Shoka Hayaki[1]

Abstract

We extend the Chan and Kogan (2002) model so that the risk attitude of representative individual is time-varying and can be negative, by using one of the ideas of prospect theory. This new behavioral asset pricing model allows some individuals facing loss to change their risk attitude from risk-averse to risk-loving. It is implied that the risk attitude of representative individual becomes counter-cyclical due to the direct effect of the business-cycle and becomes procyclical due to the indirect effect through the change in the proportion of risk-loving individuals. In addition, we provide evidence that the price of variance risk, which is associated with the risk attitude of representative individual, in major Western stock markets becomes procyclical during recessions and counter-cyclical during booms and depressions. We emphasize the following two points. First, allowing the existence of risk-loving individuals provides an economic basis for the fact that the price of variance risk is time-varying and can be negative. Second, the switch of risk attitudes at the individual level like prospect theory affects even the market level, especially during recessions. Interestingly, the indirect effect by switching risk attitudes is relatively small during the 2008 global financial crisis and the 2020 COVID-19 crisis.

Keywords: price of variance risk, risk attitude, representative individual, risk-loving, prospect theory

JEL classification: G12, G15, G40

[1] Junior Research Fellow, Research Institute for Economics and Business Administration, Kobe University / Graduate School of Business Administration, Kobe University / shoka.hayaki0817@gmail.com

1. Introduction

The capital asset pricing model (CAPM) suggests that the expectation of the market excess return from the risk-free rate is equal to the product of its variance and the "price of variance risk". In other words, the price of variance risk is equal to the ratio of the expected excess market return to its variance. The price of variance risk can be interpreted as the relative risk aversion of representative individual, according to Merton (1980). The relative risk aversion of representative individual is typically a positive constant. This property has been the rationale for assuming that the price of variance risk is also a positive constant.

It should be a strong constraint that the price of variance risk is always constant and positive. Some studies provide evidences that the price of variance risk is time-varying. In addition, some studies with the constant price of variance risk suggest different prices of variance risk for different sample periods, and the price of variance risk is not always positive.

We need a strong economic basis to claim that the price of variance risk, or the risk attitude of representative individual, is time-varying and can be negative. Chan and Kogan (2002) provided a notable economic model in which the relative risk aversion of representative individual is time-varying. Even their model, however, cannot express a negative risk attitude.

2. Time-varying price of variance risk

Chan and Kogan (2002) constructed a new continuous-time consumption-based asset pricing model that relaxes the assumptions of representative individuals in the conventional model. When an individual has the same CRRA (constant relative risk aversion) type utility function, the representative individual also has a CRRA type utility function. It is not always the case if individuals can have different CRRA type utility function. Chan and Kogan modeled such a case. In their model, individuals have various constant relative risk aversions $\gamma^{ind} \in (1, \infty)$ and the representative individual has the following utility function.

$$U^{(R)}(Y_t, X_t) = \sup_{\{c_t\}} \left\{ \int_1^{\infty} f^{(R)}(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{c_t}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \text{s. t.} \quad \int_1^{\infty} c_t \mathrm{d}\gamma^{ind} \le Y_t \right\},$$

where Y_t is an aggregate endowment, X_t is an external benchmark which represents the standard of living in the economy, $c_t = c_t(Y_t, X_t; \gamma^{ind})$ is an individual consumption, and $f^{(R)}(\gamma^{ind})$ is the distribution of individual risk attitudes.

The main implications of Chan and Kogan (2002) are summarized. First, the relative risk aversion of representative individual is expressed as $\gamma^{(R)}(Y_t, X_t) = \gamma^{(R)}(z_{Yt}) > 1$ where z_{Yt} represents the "state of the economy".[2] This implies the relative risk aversion of representative individual is the function of the state of the economy. Therefore, the risk aversion of representative individual becomes time-varying although the risk aversion of each individual is constant. Second,

[2] The state of the economy is expressed as the function of the history of aggregate endowment.

the first derivative of the relative risk aversion satisfies that $\mathrm{d}\gamma^{(R)}(z_{Yt})/\mathrm{d}z_{Yt} < 0$. The risk aversion of representative individual is counter-cyclical.

3. Prospect theory and negative price of variance risk

3.1. Risk-loving individual and prospect theory

Kahneman and Tversky (1979) and Tversky and Kahneman (1992) suggest the value function, which is concave when facing the potential gains and convex when facing the potential losses. In other words, individuals facing gains are risk-averse and individuals facing losses are risk-loving. We pay attention to such characteristics and justify the negative price of variance risk.

Chan and Kogan (2002) assume that all individuals are risk-averse. Inversely, we consider the case where all individuals become risk-loving. We suggest the utility function of risk-loving representative individual as follows.

$$U^{(L)}(Y_t, X_t) = \sup_{\{c_t\}} \left\{ \int_{-\infty}^{0} f^{(L)}(\gamma^{ind}) \frac{\lambda(\gamma^{ind})}{1-\gamma^{ind}} \left(\frac{c_t}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \mathrm{s.\,t.} \quad \int_{-\infty}^{0} c_t \mathrm{d}\gamma^{ind} \le Y_t \right\}.$$

The superscript (L) means that the model assumes a risk-loving and loss-averse individual. $\lambda(\gamma^{ind}) > 1$ reflects the property of the value function that the slope becomes steep in the loss region. Then, we can prove that the risk attitude of risk-loving representative individual has following properties.

$$\gamma^{(L)}(Y_t, X_t) \equiv \gamma^{(L)}(z_{Yt}) < 0, \qquad \mathrm{d}\gamma^{(L)}(z_{Yt})/\mathrm{d}z_{Yt} \ge 0.$$

These imply that the risk attitude of risk-loving representative individual is always negative and a decreasing function of the state of the economy, that is, procyclical. The price of variance risk will also be procyclical and always negative if all individuals in the market are risk-loving.

3.2. Mixture utility and mixture risk attitude

We consider the case where individuals with two types of risk attitudes coexist. Let $U^*(Y_t, X_t; \pi)$ be the utility function of mixture representative individual in which two types of utility functions, $U^{(R)}(Y_t, X_t)$ with $\gamma^{ind} > 1$ and $U^{(L)}(Y_t, X_t)$ with $\gamma^{ind} < 0$, are mixed using the constant π. Let f^* be the mixture distribution of individuals, which are the components of such mixture representative individual with the mixture utility function $U^*(Y_t, X_t; \pi)$. This mixture distribution is expressed as $f^*(\gamma^{ind}; \pi) = \pi f^{(R)}(\gamma^{ind}) + (1-\pi) f^{(L)}(\gamma^{ind})$. The mixture utility is represented as $U^*(Y_t, X_t; \pi) = \pi U^{(R)}(\pi Y_t, \pi X_t) + (1-\pi) U^{(L)}\big((1-\pi)Y_t, (1-\pi)X_t\big)$. Then, we can prove that the risk attitude of mixture representative individual is the function of the state of the economy, $\gamma^*(Y_t, X_t; \pi) = \gamma^*(z_{Yt}; \pi)$, and has following property.

$$\gamma^*(z_{Yt}; \pi) = \Pi(z_{Yt}; \pi)\gamma^{(R)}(z_{Yt}) + \big(1 - \Pi(z_{Yt}; \pi)\big)\gamma^{(L)}(z_{Yt}),$$

where the function $\Pi(z_{Yt}; \pi)$ is in [0,1]. It can be interpreted as the weight of $\gamma^{(R)}(z_{Yt})$. The comparative static of $\gamma^*(z;\pi)$ with respect to z is expressed below.

$$\frac{\mathrm{d}\gamma^*(z;\pi)}{\mathrm{d}z} = \Pi(z;\pi)\frac{\mathrm{d}\gamma^{(R)}(z)}{\mathrm{d}z} + \big(1-\Pi(z;\pi)\big)\frac{\mathrm{d}\gamma^{(L)}(z)}{\mathrm{d}z} + \frac{\mathrm{d}\Pi(z;\pi)}{\mathrm{d}z}\big(\gamma^{(R)}(z)-\gamma^{(L)}(z)\big).$$

The first and second term mean a mixture of first derivatives with $\Pi(z;\pi)$ and the third term is a negative adjustment term. We cannot determine the sign of the mixture terms. On the other hands, the third term is negative because we can prove that $\mathrm{d}\Pi(z;\pi)/\mathrm{d}z < 0$. Therefore, $\gamma^*(z;\pi)$ is *almost* counter-cyclical. This is the direct effect of business-cycle.

3.3. *Switching between risk-averse and risk-loving preferences*

$U^*(Y_t, X_t; \pi)$ is the utility of representative individual in a situation where the proportion of risk-averse (risk-loving) individuals is always π (always $1-\pi$). It is more natural to think that π is time-varying, however. We introduce a latent state variable S_t such that $\pi = \pi_n$ when $S_t = n$ so that the ratio of risk-averse and risk-loving individuals is *pseudo* time-varying. Then, the utility function of representative individual with the time-varying ratio of risk-averse and risk-loving individuals is defined below for a sufficiently large natural number N:

$$U(Y_t, X_t; S_t = n) = U^*(Y_t, X_t; \pi_n) \quad \text{where} \quad \pi_n \equiv n/N \quad \text{for} \quad n \in \{0,1,2,\dots,N\}$$

Prospect theory suggests that investors facing losses become risk-loving. A decline in the state of the economy z_{Yt} may increase the proportion of individuals facing losses. Therefore, a decrease in z corresponds to a decrease in π. It is natural to think that $S_t = S(z_{Yt})$, $\mathrm{d}S(z_{Yt})/\mathrm{d}z_{Yt} \geq 0$. Given $S_t = n$, the risk attitude of representative individual with time-varying latent state variable, $\gamma(Y_t, X_t; S_t = n)$, has following properties in each latent state.

$$\gamma(Y_t, X_t; S_t = n) = \gamma^*(z_{Yt}; \pi_n).$$

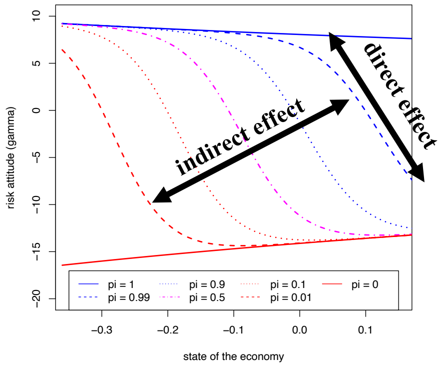
The comparative static of $\gamma^*(z;\pi)$ with respect to π is $\mathrm{d}\gamma^*(z;\pi)/\mathrm{d}\pi > 0$. This is the indirect effect of business-cycle through the π shift. Such indirect effect is procyclical. Consequently, even if $\mathrm{d}\gamma^*(z;\pi)/\mathrm{d}z$ is negative, when the increase in z and the increase in π occur at the same time, $\gamma^*(z;\pi)$ may decrease, increase, or change little because some of these effects on $\gamma^*(z;\pi)$ are offset.

Figure 1 shows the relationship between the risk attitude of representative individual $\gamma(z)$ and the state of the economy z.[3] The curves represent $\gamma^*(z;\pi=1) = \gamma^{(R)}(z)$, $\gamma^*(z;\pi=0.99)$, … , $\gamma^*(z;\pi=0) = \gamma^{(L)}(z)$. The curve of $\gamma^{(R)}(z)$ is a monotonic decreasing of z, and the curve of $\gamma^{(L)}(z)$ is a monotonic increasing of z. The curves of $\gamma^*(z;\pi=0.99)$, … are decreasing at low z and increasing at high z. It can be seen that the risk attitude $\gamma(z)$ can take any point between curve $\gamma^{(R)}(z)$ and curve $\gamma^{(L)}(z)$. The movement between the upper left and the lower right on the figure corresponds to the counter-

[3] Figure 1 suppose that $h^{(R)}(z) = -18\ln\big(0.4(z+2.55)\big) - z$, $h^{(\widetilde{L})}(z) = 36\ln\big(0.4(z+2.55)\big)$, and $F^{(\widetilde{L})} = 2.25$. See the main text for the definition of each function or parameter.

cyclicality of γ, while the movement between lower left and the upper right corresponds to the procyclicality. The time-variability of γ becomes counter-cyclical by the direct effect of the business-cycle, that is, the changes in the state of the economy. On the other hand, that becomes procyclical by the indirect effect through π shift, that is, the change changes the proportion of risk-loving individuals.

Figure 1. Structure of risk attitude and π shift



4. Empirical evidences

The function $\gamma(z)$ may have one or more inflection points by mixing the direct and indirect effect of business-cycle. For this reason, we make some polynomial approximations to capture the various relationships between the proxy state of the economy and the price of variance risk. That is, a linear model $\gamma(z) = \delta_0 + \delta_1 z$, a quadratic model $\gamma(z) = \delta_0 + \delta_1 z + \delta_2 z^2$, a cubic model $\gamma(z) = \delta_0 + \delta_1 z + \delta_2 z^2 + \delta_3 z^3$, and a quartic model $\gamma(z) = \delta_0 + \delta_1 z + \delta_2 z^2 + \delta_3 z^3 + \delta_4 z^4$. Let the model with the smallest AIC (Akaike's information criterion) be the time-varying model. Next, we use GARCH-in-mean (generalized autoregressive conditional heteroskedasticity in mean) model proposed by Engle, Lilien and Robins (1987) in order to express the discrete-time CAPM with the time-varying price of variance risk for the excess market return r^e_{Mt}:

$$r^e_{Mt} = \gamma(z_{t-1})\sigma^2_{t|t-1} + \epsilon_{Mt}, \qquad \epsilon_{Mt} \sim \mathcal{N}\left(0, \sigma^2_{t|t-1}\right).$$

We assume that the dynamics of the conditional variance follows the GARCH (1,1) process: $\sigma^2_{t|t-1} = \alpha_0 + \alpha_1 e^2_{t-1} + \beta\sigma^2_{t-1|t-2}$ where e_{t-1} is the residual at time $t-1$.

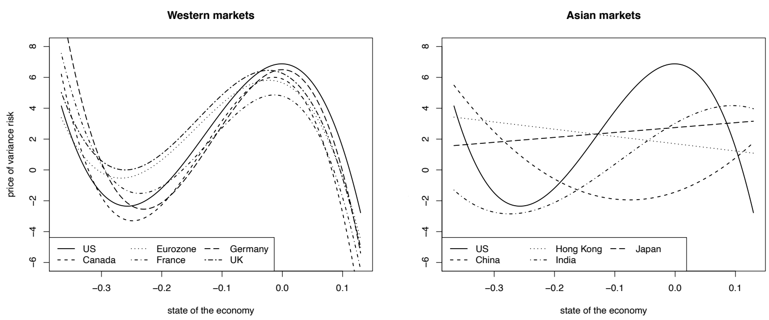
We analyze the 10 major stock markets to investigate whether the mechanism for time-variability in the price of variance risk varies from country to country.[4] The length of the sample period is 783 weeks from July 1, 2005 to June 26, 2020. Figure 2 shows the structure of risk-pricing with conditional volatility. Western stock markets have similar cubic structure.[5]

---

[4] Canada, Eurozone, France, Germany, the United Kingdom, and the United States in Western markets, and China, Hong Kong, India, Japan in Asian markets. The index of each market is used for the weekly market return; TSX, STOXX 600, CAC 40, DAX, FTSE 100, and S&P 500, and SSE, HSI, SENSEX, and TOPIX, respectively.

[5] In the Western market, the time-varying model is strongly supported from the viewpoint of the significance of the parameters and the likelihood ratio test (the null hypothesis is the constant model). On the other hand, in the Asian market, the time variability could not be approximated well by this method.

Figure 2. Structure of variance risk-pricing



We focus only on the results of the Western markets. The state of the economy at the first inflection point of each cubic function is around -0.25, and at the second inflection point is around 0. The range $[-0.25, 0]$ of the state of the economy corresponds to a recession. Figure 2 implies that the price of variance risk in Western markets are procyclical during recessions. That is, the indirect effect of the business cycle becomes stronger. On the other hand, the direct effect of the business cycle becomes stronger during other periods such as the boom and more severe recession (or depression). This is thought to be because the proportion of risk-averse individuals reaches the upper limit during the boom and reaches the lower limit during the depression.

We conclude that the indirect effect by switching risk attitudes at the individual level like prospect theory affects even the market level, especially during recessions. On the contrary, this effect weakens during booms and depressions.[6] The switch of each individual's risk attitude depends not only on whether he or she faces a loss like prospect theory, but also on the economic environment at that time. If the switch from risk-averse to risk-loving attitude is due to fear, a boom relieves fear, a recession promotes fear, and when it is full, it becomes a depression.

Reference

Chan, Y. L., and L. Kogan, 2002, "Catching up with the Joneses: Heterogeneous preferences and the dynamics of asset prices", *Journal of Political Economy* 110 (6), 1255-1285.

Engle, R. F., D. M. Lilien and R. P. Robins, 1987, "Estimating time varying risk premia in the term structure: The ARCH-M model", *Econometrica* 55 (2), 391-407.

Kahneman, D., and A. Tversky, 1979, "Prospect theory: An analysis of decision under risk", *Econometrica* 47 (2), 263-291.

Merton, R. C., 1980, "On estimating the expected return on the market: An exploratory analysis", *Journal of Financial Economics* 8, 323-361.

Tversky, A., and D. Kahneman, 1992, "Advances in prospect theory: Cumulative representation of uncertainty", *Journal of Risk and Uncertainty* 5, 297-323.

[6] The state of the economy below the first inflection point in the Western markets correspond to the 2008 global financial crisis and the 2020 COVID-19 crisis.